

# Dissipation in finite Fermi systems

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## Abstract

We present a systematic theory of dissipation in finite Fermi systems. This theory is based on the application of periodic orbit theory to linear response of many-body systems. We concentrate only on the mesoscopic aspect of the phenomena wherein a many-body system can be reduced to a single-body in an effective mean-field. We obtain semiclassical periodic-orbit corrections on top of the two-time correlation function for the rate of energy dissipation. We show that this energy dissipation is irreversible on an observational time-scale. To do so, we derive a generalised Smoluchowski equation for the energy distribution in the quantal domain. Employing the Weyl-Wigner expansion, we also obtain the equation governing the evolution of energy distribution in the combination of semiclassical and adiabatic approximations. Further, we show how the periodic orbit corrections are related to geometric phase acquired by single-particle wavefunction as it evolves with the slow, time-varying mean-field. We present our results for the important case of mixed dynamics also. We obtain random-matrix results for response functions. We incorporate chaos in the underlying classical system by writing an ansatz for a generic wavefunction which leads to various central results of equilibrium statistical mechanics. This new formalism is extended here to include dissipation. Finally, we present an expression for the viscosity tensor encountered in nuclear fission in terms of periodic orbits of single particle in an adiabatically deforming nucleus.

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## 1. Introduction

Nuclei and metallic clusters support a number of collective excitations which are well-known and well-studied [1, 2]. Damping of these collective excitations is not well understood. Dissipation related to chaotic motion of a single particle in an effective mean-field has been proposed and pursued for last two decades [3, 4]. We are interested in the mesoscopic aspect of this phenomenon where the mean free path of a single particle is of the order of the size of the system. For instance, the frequency at which an isovector giant dipole resonance is excited in a nucleus is such that the corresponding wavelength is larger than the nuclear size. In the standard parlance of nuclear physics, this is referred to as one-body dissipation. In this paper, we present a systematic semiclassical theory of one-body dissipation. Since it is well-known that several spectral properties of chaotic quantum systems, nuclei etc. are well represented by random matrix theory, we present results where random matrix theory is conjointly taken with linear response theory.

This work is inspired by some recent developments where some of the most important results of equilibrium statistical mechanics have been derived, starting from chaos in the single-particle motion in gases. Due to presence of chaos, the eigenfunctions of the corresponding quantum system become irregular, where irregularity is quantified by some correlations. For instance, spatial correlation function of an irregular eigenfunction is given by a Bessel function [5, 6]. The eigenfunctions can be modelled by a random superposition of plane waves. Remarkably, from this ansatz, without thermal reservoir, all the momentum distributions (viz., Maxwell-Boltzmann, Fermi-Dirac, Bose-Einstein) have been re-derived by Srednicki [6]. This approach plays a very important role in two dimensions where some of the standard methods of statistical mechanics fail. For the two-dimensional case, the momentum distribution was found [7] for an anyon gas upto  $\mathcal{O}(\hbar^2)$ . It was also shown [8] that the ansatz is consistent with random matrix theory. Employing a random matrix argument, the second law of thermodynamics could also be arrived at by Jain and Alonso [7]. In this work, we attempt to extend this framework to include one-body dissipation.

Let us recall that for a nucleus participating in a reaction, the time-scale is  $\sim 10^{-20}$ s or even shorter. It has been qualitatively argued [9] that in this situation, a nucleus is an isolated system. Thus, we must arrive at the notion of dissipation for an isolated quantum system since we have already succeeded in arguing for thermalisation of isolated quantum many-body systems [6].

A formal theory of one-body dissipation was developed in classic works where linear response techniques were used [11, 10]. Our considerations also rest on linear response theory. Beginning from linear response theory, we present a semiclassical expression for response function. For sake of comparison, we also present random matrix expression. From this, we obtain

the rate of energy dissipation. Clearly, if there is energy dissipation, the energy distribution must evolve irreversibly in the quantal problem. This becomes an important issue as there can be no irreversibility or dissipation in an isolated quantal system that supports only pure point spectrum [12]. Employing multiple time-scale analysis, we show that for sufficiently long times, there is irreversibility if the spectrum is coarse-grained (e.g., by a random matrix ensemble average). Beginning from the von Neumann equation for the density operator, we show that the energy distribution obeys a generalised Smoluchowski equation. When semiclassical and adiabatic approximations are put together, it is a highly singular situation. We perform a double expansion, one in the small adiabaticity parameter,  $\epsilon$ , and the other in  $\hbar$ . We then obtain the equation for energy distribution valid upto times of order  $\epsilon^{-1}$ , and upto order  $\hbar^2$ . With this derivation, we have thus found the solution of the following general problem. If a quantal system is evolving adiabatically, the energy levels of the system will evolve in a complicated manner [13]. As the levels come much closer than the mean level spacing at a particular epoch of evolution, the non-adiabatic transitions become more probable. This can be understood by employing degenerate perturbation theory. Thus, if we distribute some particles initially (at time,  $t = 0$ ) in accordance with certain law (e.g., the Boltzmann law), the final distribution will be very different. It has been thought that the evolution of the energy distribution must be diffusive. Although the guess is qualitatively correct, the equation is not the Smoluchowski equation but another equation which we have called as generalised Smoluchowski equation [14]. This general problem is encountered in many fields of physics and chemistry.

Subsequently, we present relations connecting single-particle properties and bulk properties. We show that the geometric phase acquired by a single-particle wavefunction due to adiabatically vibrating mean-field is related to the response function. Although this result has been obtained earlier [15], we briefly discuss here to put it in proper perspective.

All the above results assume that single-particle dynamics is chaotic. In realistic systems, all the symmetries are not broken. Thus the dynamics is usually mixed, i.e., there are stable islands embedded in stochastic sea. We obtain the results for this case which are based on trace formula for the case of partially broken symmetry obtained by Creagh [16].

The response function should depend on the shape of the cavity or the mean-field potential. In a relatively recent attempt, shape dependence was incorporated by multiplying by a factor,  $\overline{\mu}$ , the average fraction of trajectories which are chaotic when a uniform sampling is done [17]. This is based on some heuristic arguments. It was shown that this chaos weighted wall formula is in a better agreement with experimental data [18]. This observation and knowing that the spectral properties of non-integrable systems can be understood using semiclassical arguments based on the trace formulae [19], we believe that the theory developed here is logical and natural. Since

it is known that the spectral properties of chaotic quantum systems can be understood in terms of semiclassical trace formulae, our approach is more general and system-specific. We believe that the development presented here provides a better understanding of the interplay between single-particle motion and collective motions in a many-body quantum system.

## 2. One-body dissipation and wall formula

To understand damping in low-energy heavy-ion collisions, the concept of one-body dissipation was introduced [3] where dissipation results from the interaction of nucleons with the time-varying mean-field. This involves a reduction of many-body system to a single-particle in an effective mean-field. Although two-body collisions are also important, due to Pauli exclusion principle, the available phase space for such collisions is much lesser and the one-body mechanism is expected to dominate in the mesoscopic regime.

Assuming that the total one-body Hamiltonian is  $H(\mathbf{r}, \mathbf{p}; t)$  where  $(\mathbf{r}, \mathbf{p})$  are phase space coordinates of a single particle. Splitting  $H$  into a time-independent and a time-dependent part, respectively  $H_0(\mathbf{r}, \mathbf{p})$  and  $H_1(\mathbf{r}, \mathbf{p}; t)$ , under the assumptions of linear response theory, the rate of energy dissipation is given by [11],

$$\begin{aligned} \dot{E} = & - \int \frac{d\mathbf{r}d\mathbf{p}}{(2\pi\hbar)^3} \left[ \int_0^\infty dt' \dot{H}_1(\mathbf{R}_0(\mathbf{r}, \mathbf{p}; t'), \mathbf{P}_0(\mathbf{r}, \mathbf{p}; t'); t) \right] \\ & \dot{H}_1(\mathbf{r}, \mathbf{p}; t) \frac{\partial f_0}{\partial H_0}(\mathbf{r}, \mathbf{p}), \end{aligned} \quad (1)$$

where  $f_0$  is the single-particle phase space distribution and an overdot represents a time-derivative. The important quantity in (1) is the two-time correlation function in the integrand.

The rate of energy dissipation for the case when the dynamics of a single-particle is chaotic is given by the wall formula [4] :

$$\dot{E}_{\text{wall}} = \rho \bar{v} \int u^2(a) d^2a, \quad (2)$$

where  $u(a)$  is the normal component of the surface velocity at the point  $a$  on the surface,  $\rho$  and  $\bar{v}$  are nuclear mass density and average nucleon speed inside the nucleus.

The above expressions hold when the dynamics of a single particle is fully chaotic. However, since some of the symmetries are not broken, the dynamics is expected to be mixed rather than chaotic. For this case, a scaled wall formula [17],

$$\dot{E} = \bar{\mu} \dot{E}_{\text{wall}}, \quad (3)$$

has been proposed. The factor,  $\bar{\mu}$ , is the average fraction of the trajectories which are chaotic when a uniform sampling of the surface is done. The factor

is fixed from the parameter appearing in the Berry-Robnik distribution [20], which gives the nearest-neighbour level spacing distribution for systems with mixed phase space. This way of incorporating the role of dynamics is empirical. In our framework, we take care of the dynamical features explicitly, hence any role of dynamics enters explicitly. Importantly, since we employ the trace formula, there is harmony between the dynamics and the spectral statistics.

Let us now comment on the relation between chaos and dissipation. From a number of studies, classical [21] and quantal [22], it is clear that if the dynamics is chaotic, there is dissipation. If the particle and wall motions are considered self-consistently, then it has been numerically seen that the dynamics is chaotic [23]. In this case, the energy is gained irreversibly, leading to dissipation. If the dynamics is regular or integrable, the energy gained by a particle from the wall will eventually be fed back to the wall, resulting in no dissipation.

The one-body mechanism is dominant for low-energy nucleus-nucleus collisions or fission, i.e., in general, to slow collective processes. It is known that to simultaneously reproduce both the fission probability and the pre-scission neutron multiplicity, one needs a shape-dependent friction [24, 25]. These works came up as an attempt to combine dynamics and thermodynamics in a consistent way. The shape of the nucleus is reflected in the level density and this is used, in turn, to construct entropy. At this point, whereas these works in nuclear physics accept a stochastic description in working with Langevin equation, we would persevere with the dynamical considerations. We believe that friction or viscosity, if any, in these quantum systems will have quantal signatures. For instance, the response function of a many-body Fermi system is related to geometric phase acquired by a single-particle eigenfunction as the system deforms [15]. These considerations need a systematic theory where shape plays an important role. To do so, we believe that it is important to employ the Gutzwiller [26] trace formula and evolve a framework.

### 3. Quantum chaos and equilibrium statistical mechanics

The connection between chaos and equilibration or thermalization of many-body systems has been always believed in. The tenets of equilibrium statistical mechanics rest on the assumption of “molecular chaos” - an idea that goes back to Boltzmann [27]. With the advancement of our understanding of the theory of dynamical systems and its relation with statistical mechanics [28], it has become important to extend this understanding to quantal many-body systems. Only recently, such a connection has been established. Since this connection and the present approach is an underlying theme of this paper, we briefly present the status of our understanding for

want of a better perspective.

Let us mention the numerical results on eigenfunctions of systems whose classical counterparts are chaotic.

(**N1**) The amplitude distribution of eigenfunctions are found to agree with a Gaussian distribution - we call these states as the generic ones [29]. Of course, we are not referring to eigenfunctions corresponding to the ground state which is special. Nevertheless, this observation is more general as the Gaussian distribution remains good even for pseudointegrable (non-integrable and non-chaotic) systems like a  $\pi/3$ -rhombus billiard [30, 31] where the Kolmogorov-Sinai entropy is zero.

(**N2**) The spatial correlation function,

$$C_\alpha(\vec{s}) = \frac{1}{V} \int d\mathbf{r} \psi_\alpha^* \left( \mathbf{r} - \frac{\mathbf{s}}{2} \right) \psi_\alpha \left( \mathbf{r} + \frac{\mathbf{s}}{2} \right), \quad (4)$$

corresponding to an eigenfunction  $\psi_\alpha(\mathbf{r})$  is found to agree with a (cylindrical) Bessel function [5], where  $V$  is volume. For two-dimensional billiards, it has been found to agree with  $J_0(k\mathbf{s})$ ,  $k = \sqrt{2mE/\hbar^2}$  [29].

(**N3**) The nodal lines are quite complicated, for pseudointegrable as well as chaotic billiards [29, 31].

As far as eigenvalues or the energy levels are concerned, the measures quantifying spectral statistics agree with the results known from random matrix theory [32]. Even for pseudointegrable billiards, which are non-integrable, the nearest-neighbour level spacing distribution can be explained in terms of new ensembles [33].

The numerical results (**N1,N2**) can be explained if we assume that the complicated eigenfunctions are written as a random superposition of plane waves. This has been advocated by Berry [5] and written explicitly by Srednicki [6]. For an  $N$ -particle system, the canonical pair of coordinates are  $(\mathbf{X}, \mathbf{P})$  where  $\mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N)$  and  $\mathbf{P} = (\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_N)$ . The energy eigenfunction  $\psi_\alpha(\mathbf{X})$  is thus written as

$$\psi_\alpha(\mathbf{X}) = N_\alpha \int d^{dN} \mathbf{P} A_\alpha(\mathbf{P}) \delta(P^2 - 2mE_\alpha) e^{\frac{i}{\hbar} \mathbf{X} \cdot \mathbf{P}} \quad (5)$$

for a  $d$ -dimensional system, where  $N_\alpha$  is the normalization constant and the amplitudes satisfy the two-point correlation function,

$$\langle A_\alpha^*(\mathbf{P}) A_\alpha(\mathbf{P}') \rangle_{ME} = \delta_{\alpha\gamma} \frac{\delta^{dN}(\mathbf{P} - \mathbf{P}')}{\delta(P^2 - P'^2)}. \quad (6)$$

This correlator has been shown to be consistent with random matrix theory, hence the appearance of a statistical average on the left side of the equation ( $ME$  stands for a matrix-ensemble average) [8]. Beginning from this ansatz for the eigenfunction, the momentum distributions for ideal

gases have been shown to concur with the well-known results (like Maxwell-Boltzmann, Fermi-Dirac, Bose-Einstein, or fractional statistics) by Srednicki [6], and, Jain and Alonso [7]. Even the first few virial coefficients have been calculated. The Wigner distribution corresponding to an eigenfunction is microcanonical. Thus, generically, corresponding to an eigenfunction, there is entire energy surface in phase space.

An important question - in this theory, what is temperature ? Temperature quantifies heat, and heat is defined as a mode of motion. Even if there is no reservoir, one can still consistently use the kinetic theory, and write for a system with  $f$  freedoms at energy  $E_\alpha$ , a temperature,  $T_\alpha$ , with  $E_\alpha = \frac{1}{2}fkT_\alpha$ , where  $k$  is the Boltzmann constant. As the average over the entire system involves an averaging over the level density, the average temperature,  $T$  will automatically get defined. This average temperature is the one we measure as we obtain the correct momentum distributions with this temperature playing exactly the same role as the “usual” temperature.

For the case of a many-body system like a nucleus where a nucleon trajectory is chaotic, a typical eigenfunction will be written as above. Thus, as the system evolves and shape of the nucleus changes, the eigenfunction will acquire a geometric phase (see Sec. 8). It is this phase which will eventually find a relation with the absorptive part of generalised susceptibility [15].

#### 4. Semiclassical linear response

To study the effect of external perturbation on a many-body system, one can make usage of linear response theory [34, 35, 60]. Also, one can imagine a fictitious external field to study collective excitations as if they were generated due to a hypothetic field [10]. In this section, we obtain an expression for generalised susceptibility and the frequency-dependent response function (also called as the polarisation propagator [36]) in terms of periodic orbits employing the Gutzwiller trace formula [26]. We assume that the dynamics of a single particle is fully chaotic. As mentioned earlier, the important case of mixed dynamics will be taken up in Section 6. The assumption of chaos is consistent with some recent numerical investigations for the case of nuclei [23].

The system is described by a Hamiltonian  $\hat{H}$  which is disturbed by a field,  $F^{\text{ext}}(t)$ . The total Hamiltonian is

$$\hat{H}_T = \hat{H} - \hat{Q}F^{\text{ext}}(t) \quad (7)$$

where  $\hat{Q}$  is an observable, an example could be magnetization in the context of spin systems, or, an electric dipole operator in photoabsorption experiments. The response function can be written as the imaginary part of the dynamical susceptibility,

$$\chi''(t, t') = \frac{1}{2\hbar} \langle [\hat{Q}(t), \hat{Q}(t')] \rangle$$

$$= \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \tilde{\chi}''(\omega). \quad (8)$$

The angular brackets denote the expectation value and the square brackets denote the commutator. Setting an initial time to zero and final time to  $t$ , we can re-write

$$\chi''(t) = \frac{1}{2\hbar} \langle [\hat{Q}(0), \hat{Q}(t)] \rangle \quad (9)$$

where

$$\hat{Q}(t) = e^{\frac{i}{\hbar} \hat{H} t} \hat{Q} e^{-\frac{i}{\hbar} \hat{H} t}, \quad (10)$$

$\langle \dots \rangle$  denotes an average over the initial state of the system which is, for instance, the thermal state wherein

$$\langle \dots \rangle = \frac{1}{Z(\beta)} \text{tr} e^{-\beta \hat{H}} (\dots) \quad (11)$$

where the temperature,  $T = \frac{1}{k\beta}$ .

Denoting the many-body eigenstates by  $\Phi_n$  and one-body eigenstates by  $\phi_n$ , and after reducing the description of the many-body system to one-body, the propagator is

$$\tilde{\chi}''(\omega) = \sum_{a,b} |\langle \phi_a | \hat{q} | \phi_b \rangle|^2 \Im \frac{p^{FD}(\epsilon_a) - p^{FD}(\epsilon_b)}{\hbar\omega - \epsilon_a + \epsilon_b + i0^+}, \quad (12)$$

where

$$p^{FD}(\epsilon_a) = \frac{1}{e^{\beta(\epsilon_a - \mu)} + 1} \quad (13)$$

denotes the Fermi-Dirac probability of occupation number with  $\mu$  as the chemical potential. Eq. (12) is the Fourier transform of the two-time correlation function. In (12), the operator  $\hat{q}$  is the one-body operator which when taken for each body and direct-summed gives us the operator,  $\hat{Q}$ .

Now that the susceptibility has been reduced to a 1-body expression, we can treat the 1-body system semiclassically in terms of periodic orbits in an effective Hartree-Fock (or some other mean-field) Hamiltonian,  $\hat{h}$ . For subsequent convenience, let us re-write (12) as

$$\begin{aligned} \tilde{\chi}''(\omega) &= \sum_{a,b} |\langle \phi_a | \hat{q} | \phi_b \rangle|^2 [p^{FD}(\epsilon_a) - p^{FD}(\epsilon_a + \hbar\omega)] \delta(\hbar\omega + \epsilon_a - \epsilon_b) \\ &= -\pi \sum_{n=1}^{\infty} \frac{(\hbar\omega)^n}{n!} \sum_{a,b} \frac{\partial^n p^{FD}}{\partial \epsilon^n}(\epsilon_a) |\langle \phi_a | \hat{q} | \phi_b \rangle|^2 \delta(\hbar\omega + \epsilon_a - \epsilon_b) \\ &\equiv -\frac{1}{2\hbar} \sum_{n=1}^{\infty} \frac{(\hbar\omega)^n}{n!} \tilde{f}_n(\omega). \end{aligned} \quad (14)$$



Here we have introduced the time correlation function  $f_n(t)$  and its Fourier transform,  $\tilde{f}_n(\omega)$ :

$$\begin{aligned} f_n(t) &= \text{tr} \frac{\partial^n p^{FD}}{\partial \epsilon^n}(\hat{h}) e^{\frac{i}{\hbar} t \hat{h}} \hat{q} e^{-\frac{i}{\hbar} t \hat{h}} \hat{q}; \\ \tilde{f}_n(\omega) &= \int dt e^{i\omega t} f_n(t) \\ &= 2\pi\hbar \sum_{a,b} \frac{\partial^n p^{FD}}{\partial \epsilon^n}(\epsilon_a) |\langle \phi_a | \hat{q} | \phi_b \rangle|^2 \delta(\hbar\omega + \epsilon_a - \epsilon_b). \end{aligned} \quad (15)$$

In fact,

$$f_n(t) = \int d\epsilon \frac{\partial^n p^{FD}}{\partial \epsilon^n}(\epsilon) C(\epsilon, t) \quad (16)$$

with

$$C(\epsilon, t) = \text{tr} \delta(\epsilon - \hat{h}) \hat{q}(t) \hat{q}(0). \quad (17)$$

Finally, the propagator is

$$\tilde{\chi}''(\omega) = -\pi \sum_{n=1}^{\infty} \frac{(\hbar\omega)^n}{n!} \int \frac{dt d\epsilon}{2\pi\hbar} e^{i\omega t} \frac{\partial^n p^{FD}}{\partial \epsilon^n}(\epsilon) C(\epsilon, t). \quad (18)$$

The classical dynamics in the three-dimensional one-body effective potential of a fermionic system will, in general, be chaotic. Now we apply the Gutzwiller trace formula to obtain an expression for the propagator. For this, we need the semiclassical expression for the time correlation,  $C(\epsilon, t)$ , which we consider in the form :

$$\begin{aligned} C(\epsilon, t) &= \text{tr} \delta(\epsilon - \hat{h}) \hat{X} \\ &= \text{tr} \delta(\epsilon - \hat{h}) \hat{A} + i \text{tr} \delta(\epsilon - \hat{h}) \hat{B}, \end{aligned} \quad (19)$$

where  $\hat{X} = \hat{X}(t) = \hat{q}(t) \hat{q}(0)$  is one-body operator. This operator is non-hermitian but can be decomposed as  $\hat{X} = \hat{A} + i\hat{B}$  in terms of two hermitian operators :

$$\hat{A} = \frac{\hat{X} + \hat{X}^\dagger}{2}, \quad \hat{B} = \frac{\hat{X} - \hat{X}^\dagger}{2i}. \quad (20)$$

We need to re-express matrix elements of  $\hat{A}$  (and  $\hat{B}$ ) over the eigenstates of  $\hat{h}$ . These matrix elements can be obtained by employing first-order perturbation theory for the perturbed Hamiltonian  $\hat{h}(\lambda) = \hat{h} + \lambda \hat{A}$ . Assuming the eigenvalue problem for  $\hat{h}(\lambda)$  to be solved, the matrix elements of  $\hat{A}$  may thus be obtained in terms of derivatives of eigenvalues of the perturbed Hamiltonian,  $\epsilon_n(\lambda)$ , with respect to  $\lambda$ .

Each term of the correlation function  $C(\epsilon, t)$  can be expressed as

$$\begin{aligned} \text{tr } \delta(\epsilon - \hat{h})\hat{A} &= -\frac{1}{\pi} \Im \text{tr} \frac{\hat{A}}{\epsilon - \hat{h} + i0^+} \\ &= \frac{1}{\pi} \Im \text{tr} \frac{\partial}{\partial \lambda} \log(\epsilon - \hat{h} - \lambda \hat{A} + i0^+) \Big|_{\lambda=0} \end{aligned} \quad (21)$$

On comparing with the identity,

$$\begin{aligned} -\frac{1}{\pi} \frac{\partial}{\partial \epsilon} \Im \text{tr} \log(\epsilon - \hat{h} - \lambda \hat{A} + i0^+) &= \text{tr } \delta(\epsilon - \hat{h} - \lambda \hat{A}) \\ &= \frac{\partial N}{\partial \epsilon}(\epsilon; \lambda), \end{aligned} \quad (22)$$

we have

$$\text{tr } \delta(\epsilon - \hat{h})\hat{A} = -\frac{\partial N}{\partial \lambda}(\epsilon; \lambda) \Big|_{\lambda=0}, \quad (23)$$

where  $N(\epsilon; \lambda)$  is the cumulative density of levels for the parametrised Hamiltonian  $\hat{h}(\lambda)$ .

For the case under consideration where the single-particle dynamics is chaotic in an effective mean-field whereupon we can assume that the periodic orbits are isolated and unstable, the cumulative density of levels is given by the well-known expression due to Gutzwiller [26],

$$\begin{aligned} N(\epsilon; \lambda) &= \int \frac{d^f x d^f p}{(2\pi\hbar)^f} \Theta[\epsilon - h_W(\lambda)] + \mathcal{O}(\hbar^{-f+1}) \\ &+ \sum_p \sum_{r=1}^{\infty} \frac{1}{r\pi} \frac{\sin \left[ \frac{r}{\hbar} S_p(\epsilon; \lambda) - r \frac{\pi}{2} \nu_p \right]}{|\det(\mathbf{m}_p^r(\lambda) - \mathbf{I})|^{\frac{1}{2}}} + \mathcal{O}(\hbar). \end{aligned} \quad (24)$$

Here,  $\mathbf{m}_p^r(\lambda)$  is the monodromy matrix governing the stability of the classical periodic orbits,  $p$ ;  $\nu_p$  denotes the Maslov index of the trajectory, and  $h_W$  is the Weyl symbol of the Hamiltonian operator,  $\hat{h}(\lambda)$ .

Owing to (24), we need the derivative of the action w.r.t. the parameter,  $\lambda$  [37]:

$$\frac{\partial S_p}{\partial \lambda} = - \oint dt \frac{\partial h_W(\lambda)}{\partial \lambda} = - \oint_p dt A_W \quad (25)$$

where  $A_W$  is the Weyl symbol of  $\hat{A}$ . Thus,

$$\begin{aligned} \text{tr } \delta(\epsilon - \hat{h})\hat{X} &= \int \frac{d^f x d^f p}{(2\pi\hbar)^f} X_W \delta[\epsilon - h_{\text{cl}}] + \mathcal{O}(\hbar^{-f+1}) \\ &+ \frac{1}{\pi\hbar} \sum_p \sum_{r=1}^{\infty} \frac{\cos \left[ \frac{r}{\hbar} S_p - r \frac{\pi}{2} \nu_p \right]}{|\det(\mathbf{m}_p^r(\lambda) - \mathbf{I})|^{\frac{1}{2}}} \oint_p dt X_W + \mathcal{O}(\hbar^0), \end{aligned} \quad (26)$$

where  $X_W$  is the Weyl symbol of  $\hat{X}$ .

For our case where

$$\hat{X} = \exp\left(\frac{i}{\hbar}\hat{h}t\right)\hat{q}\exp\left(-\frac{i}{\hbar}\hat{h}t\right)\hat{q}, \quad (27)$$

the Weyl symbol can be written as [38]

$$\begin{aligned} X_W(\mathbf{x}, \mathbf{p}) &= (e^{\frac{i}{\hbar}\hat{h}t})_W e^{\frac{i}{2}\hbar\Lambda} \left\{ q_W e^{\frac{i}{2}\hbar\Lambda} \left[ (e^{-\frac{i}{\hbar}\hat{h}t})_W e^{\frac{i}{2}\hbar\Lambda} q_W \right] \right\} \\ &= \left[ \exp(-\hat{\mathcal{L}}_{cl}t) \right] q_W + \text{Weyl-Wigner corrections} \\ &= q_W(t)q_W(0). \end{aligned} \quad (28)$$

The operator,  $\hat{\mathcal{L}}_{cl}$  is the classical Liouvillian operator defined by  $\{h, \cdot\}$ , and the operator  $\hat{\Lambda}$  is defined by

$$\hat{\Lambda} = \frac{\overleftarrow{\partial}}{\partial \mathbf{p}} \cdot \frac{\overrightarrow{\partial}}{\partial \mathbf{x}} - \frac{\overleftarrow{\partial}}{\partial \mathbf{x}} \cdot \frac{\overrightarrow{\partial}}{\partial \mathbf{p}}, \quad (29)$$

and  $q_W(t) = q_W[\Phi^t(\mathbf{x}, \mathbf{p})]$  with the Hamiltonian flow denoted by  $\Phi^t$ .

Finally, the correlation function is

$$\begin{aligned} C(\epsilon, t) &= \text{tr } \delta(\epsilon - \hat{h})\hat{q}(t)\hat{q}(0) \\ &= \int \frac{d^f x d^f p}{(2\pi\hbar)^f} q_W \left[ \exp(-\hat{\mathcal{L}}_{cl}t) \right] q_W \delta[\epsilon - h_{cl}] + \mathcal{O}(\hbar^{-f+1}) \\ &+ \frac{1}{\pi\hbar} \sum_p \sum_{r=1}^{\infty} \frac{\cos \left[ \frac{r}{\hbar} S_p - r \frac{\pi}{2} \nu_p \right]}{|\det(\mathbf{m}_p^r(\lambda) - \mathbf{I})|^{\frac{1}{2}}} \oint_p d\tau q_W(\tau) q_W(t + \tau) \\ &+ \mathcal{O}(\hbar^0). \end{aligned} \quad (30)$$

The propagator or frequency-dependent response function is given by

$$\begin{aligned} \tilde{\chi}''(\omega) &= \frac{1}{\hbar} \int dt \int \frac{d^f x d^f p}{(2\pi\hbar)^f} \left[ p^{FD}(\epsilon) - p^{FD}(\epsilon + \frac{\hbar\omega}{2}) \right] q_W(\mathbf{x}, \mathbf{p}) \\ &\quad \left[ e^{(i\omega - \hat{\mathcal{L}}_{cl})t} q_W \right](\mathbf{x}, \mathbf{p}) + \frac{1}{\pi\hbar^2} \int d\epsilon \left[ p^{FD}(\epsilon) - p^{FD}(\epsilon + \frac{\hbar\omega}{2}) \right] \\ &\quad \sum_p \sum_{r=1}^{\infty} \frac{\cos \left[ \frac{r}{\hbar} S_p - r \frac{\pi}{2} \nu_p \right]}{|\det(\mathbf{m}_p^r(\lambda) - \mathbf{I})|^{\frac{1}{2}}} \int dt e^{i\omega t} \oint_p d\tau q_W(\tau) q_W(t + \tau) \end{aligned} \quad (31)$$

Since the rate of energy dissipation is directly related to  $\tilde{\chi}''(\omega)$ , the above expression gives us a semiclassical description of dissipation. Note that the shape (which means potential) of a system dictates the periodic orbits and the Liouvillian operator.

## 5. Random-matrix linear response

Energy level sequences of nuclei have local fluctuation properties which fit remarkably well with the predictions of random matrix theory [39]. So is

the case with other chaotic quantum systems [32]. An attempt to arrive at a connection between random matrix universality and chaos has been made recently [41]. In this section, we present random-matrix expression for linear response for Fermi systems. Thus the Fermi system is one in an ensemble, where the Hamiltonian matrix is modelled in terms of a random matrix. In the context of absorption by small metallic particles, random matrix theory was used by Gorkov and Eliashberg [42].

Given a level sequence,  $\{x_i\}$ , the  $n$ -level correlation function is defined as [40]

$$R_n(x_1, x_2, \dots, x_n) = \frac{N!}{(N-n)!} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dx_{n+1} \dots dx_N P_N(x_1, x_2, \dots, x_N) \quad (32)$$

which is the probability density of finding a level (regardless of labelling) around each of the points  $x_1, x_2, \dots, x_n$ , the positions of remaining levels being unobserved.  $P(x_1, \dots, x_N)$  denotes the joint probability distribution function for the levels  $x_1, \dots, x_N$ . The level density is given by the one-point function,  $R_1(x)$ . The  $n$ -level cluster function is defined as

$$T_n(x_1, x_2, \dots, x_n) = \sum_{\mathbf{G}} (-1)^{n-m} (m-1)! \prod_{j=1}^m R_{G_j}(x_k, \text{with } x_k \in G_j), \quad (33)$$

where  $\mathbf{G}$  stands for any division of the indices  $(1, 2, \dots, n)$  into  $m$  subgroups  $(G_1, G_2, \dots, G_m)$ . For instance,

$$\begin{aligned} T_1(x) &= R_1(x), \\ T_2(x_1, x_2) &= -R_2(x_1, x_2) + R_1(x_1)R_1(x_2), \end{aligned} \quad (34)$$

and so on. Since we are only interested in the local fluctuations, the energy levels  $x_i$  and  $x_j$  are such that  $N|x_i - x_j| \sim \mathcal{O}(1)$ , the so-called scaling limit. In the scaling limit, the two-level cluster function,  $T_2(x_1, x_2)$ , becomes  $Y_2(|x_1 - x_2|)$ .

Time correlations will now be averaged over the ensemble of random matrices which entail an ensemble-averaged time correlation function. Each term is multiplied by the probability that two levels will be in the intervals  $d\epsilon_1$  and  $d\epsilon_2$ ,

$$Y_2(|\epsilon_1 - \epsilon_2|) \frac{d\epsilon_1}{\bar{\sigma}} \frac{d\epsilon_2}{\bar{\sigma}} = Y_2(|\epsilon_1 - \epsilon_2|) n_{TF}^2 d\epsilon_1 d\epsilon_2 \quad (35)$$

where  $\bar{\sigma}$  is the mean level spacing and  $n_{TF}$  is the Thomas-Fermi density of levels. We get for the response function :

$$\begin{aligned} \tilde{\chi}''(\omega) &= n_{TF}^2 \sum_{m=1}^{\infty} \frac{(\hbar\omega)^m}{m!} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\epsilon_1 d\epsilon_2 \frac{\partial^m p^{FD}(\epsilon)}{\partial \epsilon^m} \delta(\hbar\omega - \epsilon_2 + \epsilon_1) \\ &\quad Y_2(|\epsilon_1 - \epsilon_2|) |\langle \phi_1 | \hat{q} | \phi_2 \rangle|^2. \end{aligned} \quad (36)$$

For the invariant ensembles, i.e., the orthogonal, the unitary, or the symplectic ensembles,  $Y_2(|\epsilon_1 - \epsilon_2|)$  embodies universal results. Thus the random matrix expression gives the generic form. It must be borne in mind that the local fluctuations and universalities have been qualitatively (and sometimes quantitatively) understood using semiclassical methods. The random matrix result provides us a trend, a qualitative overall behaviour; it is the semiclassical theory containing microscopic information that can eventually give us system-specific results.

The matrix element appearing in (36) can be related to the classical autocorrelation function of  $q_{cl}$  (the classical counterpart of  $\hat{q}$ ) if we assume that the sum of mean values of any quantity over all states with definite energy roughly equals the sum of the classical mean values of this quantity as the particle moves over all the trajectories with given energy [50].

Thus,

$$\begin{aligned}\tilde{\chi}''(\omega) &= \frac{n_{TF}}{2\pi} \sum_{m=1}^{\infty} \frac{(\hbar\omega)^m}{m!} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\epsilon_1 d\epsilon_2 \frac{\partial^m p^{FD}(\epsilon)}{\partial \epsilon^m} \delta(\hbar\omega - \epsilon_2 + \epsilon_1) \\ &\quad Y_2(|\epsilon_1 - \epsilon_2|) \int_{-\infty}^{\infty} d\tau e^{\frac{i}{\hbar}(\epsilon_2 - \epsilon_1)\tau} C_{qq}(\tau)\end{aligned}\quad (37)$$

where the correlation function is

$$C_{qq}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt q(t) q(t + \tau). \quad (38)$$

Changing the variables to

$$\begin{aligned}\epsilon_- &= \epsilon_2 - \epsilon_1 \\ \epsilon_+ &= \frac{\epsilon_2 + \epsilon_1}{2}\end{aligned}\quad (39)$$

and after some manipulations, we get a compact result :

$$\tilde{\chi}''(\omega) = n_{TF} \frac{\hbar\omega}{2} Y_2(\hbar\omega) S_{qq}(\omega). \quad (40)$$

In this expression,  $S_{qq}(\omega)$  is the spectral density defined by

$$S_{qq}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} C_{qq}(\tau). \quad (41)$$

For different Gaussian ensembles [40] and non-Gaussian, invariant ensembles [43], the two-point function is

$$\begin{aligned}Y_2(\hbar\omega) &= \left[ \frac{\sin(\pi\hbar\omega)}{\pi\hbar\omega} \right]^2 \\ &= \left[ \frac{\sin(\pi\hbar\omega)}{\pi\hbar\omega} \right]^2 + \left[ \int_{\hbar\omega}^{\infty} \left( \frac{\sin(\pi t)}{\pi t} \right) dt \right] \left[ \frac{d}{d(\hbar\omega)} \left( \frac{\sin(\pi t)}{\pi t} \right) \right] \\ &= \left[ \frac{\sin(2\pi\hbar\omega)}{2\pi\hbar\omega} \right]^2 - \frac{d}{d(\hbar\omega)} \left[ \frac{\sin(2\pi\hbar\omega)}{2\pi\hbar\omega} \right] \int_0^{\hbar\omega} \left[ \frac{\sin(2\pi t)}{2\pi t} \right] dt\end{aligned}\quad (42)$$

corresponding to unitary, orthogonal, and symplectic ensembles respectively.

For the case when a random matrix,  $h$ , is perturbed by another random matrix,  $q$ , independent from  $h$  but invariant with respect to the same symmetry,  $S_{qq}(\omega)$  is explicitly known [53]. The spectral density becomes asymmetric at non-zero temperatures and shows a universal dip at zero frequency. The behaviour of the spectral density near zero frequency is universal, depending on the symmetry parameter.

## 6. Response of mixed systems

The most common situation is when the dynamics is neither fully integrable nor fully chaotic, but mixed. Looking at the phase space of such systems, we observe stable islands embedded in stochastic sea. We present an expression for the response function employing the recent semiclassical result for mixed systems [16].

The semiclassical expression for the density of energy levels is given by the trace of the Green function in position representation, and it results in a sum over periodic orbits. The integration in the method of stationary phase goes from one point  $\mathbf{x}$  to itself collecting contributions of periodic orbits. Here the symmetry becomes important. If there is a continuous symmetry, then there is a family of periodic orbits with given energy. Only the integration transverse to this family can be done by the method of stationary phase, the parallel integration remains. Following [16], let us assume that the symmetry group is  $G$ . In case  $G$  acts on the phase space in a way that infinitesimal generators of the group, and the vector field of the Hamiltonian itself, are all linearly independent, the subgroup of  $G$  which leaves the periodic orbits invariant should be discrete. In this case, the degeneracy of periodic orbits,  $k$  is just the dimension of the group. Also, if  $\gamma_0(t)$  is a reference orbit, other orbits can be parametrised by group elements  $g$  according to

$$\gamma_g(t) = g\gamma_0(t). \quad (43)$$

The points on the family are parametrised by  $(g, t)$  and the  $(k+1)$ -dimensional measure is  $dt d\mu(g)$  where  $d\mu(g)$  is the Haar measure [44]. The trace formula for the case of exact symmetry becomes [45],

$$d(E) = \frac{1}{i\hbar} \frac{1}{(2\pi i\hbar)^{k/2}} \sum_{p \in \Gamma} \int dt d\mu(g) |K|^{-\frac{1}{2}} e^{\frac{iS(E)}{\hbar} - \frac{i\sigma\pi}{2}} \quad (44)$$

where  $K$  is an invariant of the family and is determined by linearisation of dynamics about a typical orbit.

For nontrivial symmetry,

$$K = Q \det W \det(\tilde{M} - I). \quad (45)$$

Here  $Q$  is the jacobian, independent of dynamics from the choice of basis for the Lie algebra,  $W$  is the symplectic matrix from the linearisation in full phase space of the dynamics around the orbit, and tilde on  $M$  represents the fact that symmetry has been extracted. In (44),  $S(E)$  is the action of a typical orbit in the family, and,  $\sigma = \mu - \delta$  where  $\mu$  is the Maslov index and  $\delta$  is the number of positive eigenvalues of  $W$ .

Now we perturb a system (with Hamiltonian  $H_0$ ) with continuous group  $G$  so that the resulting Hamiltonian,  $H$  has no symmetry at all,

$$H = H_0 + \lambda H_1. \quad (46)$$

For this case, the trace formula becomes [16]

$$d(E; \lambda) = \frac{1}{i\hbar} \frac{1}{(2\pi i\hbar)^{k/2}} \sum_{po, \Gamma} \int_{\Gamma} dt d\mu(g) |K|^{-\frac{1}{2}} e^{\frac{iS_0(E)}{\hbar} - \frac{i\sigma\pi}{2}} \langle e^{i\frac{\lambda}{\hbar} F(g, E)} \rangle_{g \in G}, \quad (47)$$

where

$$F(g, E) = - \int_{\gamma_g(t)} dt H_1 = \frac{\Delta S}{\lambda}(\mathbf{x}_{||}, E; \lambda). \quad (48)$$

The angular bracket represents an average over the orbit label  $g$ .  $S_0$  is the action of a trajectory of unperturbed Hamiltonian and  $\Delta S$  is a correction of order  $\lambda$ . To provide an expression for two-time correlation function, we first note, progressing in the same way as in Sec. 4,

$$\begin{aligned} \text{tr } \delta(E - \hat{H}) \hat{X} &= \int \frac{d^f x d^f p}{(2\pi\hbar)^f} X_W \delta(E - H_{cl}) + \mathcal{O}(\hbar^{-f+1}) + \frac{1}{i\hbar} \frac{1}{(2\pi i\hbar)^{k/2}} \\ &\quad \sum_{po, \Gamma} \int_{\Gamma} dt d\mu(g) |K|^{-\frac{1}{2}} e^{\frac{iS_0(E)}{\hbar} - \frac{i\sigma\pi}{2}} \frac{\partial \mathcal{M}}{\partial \lambda} \left( \frac{\lambda}{\hbar}, E \right) \end{aligned} \quad (49)$$

where

$$\mathcal{M} \left( \frac{\lambda}{\hbar}, E \right) = \langle e^{i\frac{\lambda}{\hbar} F(g, E)} \rangle_{g \in G}, \quad (50)$$

and  $\hat{X}$  is defined in Sec. 4. In general, let  $\hat{X}$  be  $\hat{A}(0)\hat{B}(t)$ . For the case of mixed dynamics, we obtain the following expression where the system is in thermal equilibrium at temperature,  $T$ :

$$\begin{aligned} C_{AB}(t, \lambda, T) &= \frac{1}{Z(\beta)} \int \frac{d^f x d^f p}{(2\pi\hbar)^f} e^{-\beta H_{cl}} A_W(x, p) e^{-\mathcal{L}_{cl} t} B_W(x, p) \\ &+ \mathcal{O}(\hbar^{-f+1}) + \frac{1}{i\hbar} \frac{1}{(2\pi i\hbar)^{k/2}} \\ &\quad \frac{1}{Z(\beta)} \sum_{po, \Gamma} \int dE e^{-\beta E} \int_t^{t+T_{po}} dt' d\mu(g) |K|^{-\frac{1}{2}} \\ &\quad e^{\frac{iS_0(E)}{\hbar} - \frac{i\sigma\pi}{2}} \frac{\partial}{\partial E} \left\langle \exp \left[ -\frac{i\lambda}{\hbar} \int_{\gamma_g(\tau)} d\tau A(\tau) B(\tau + t') \right] \right\rangle_{g \in G} \end{aligned} \quad (51)$$

Once the group  $G$  is identified, the calculation of  $\langle\!\rangle_{g\in G}$  is not difficult. For the case of axial symmetry, it has been demonstrated in [16]. In the case of nuclear physics, the connection between deformed nuclei and harmonic oscillators where the frequency ratios can be related to deformation parameters, a much desirable simplification is expected to occur.

Note that the decay of correlations will be governed by the eigenvalues of the classical Liouvillian operator. For the systems which are mixing, the decay of correlations is exponential. For mixed systems, this decay will be more complicated and system-specific. The important point is that dynamical aspects have been incorporated explicitly. We believe that it will lead us to an understanding of many important aspects of many-body systems, pre-fission neutron multiplicity, damping of giant resonances being some of the examples.

## 7. Irreversibility of energy dissipation

It has been argued in the past that the irreversibility of energy dissipation occurs when the single particle dynamics is chaotic [9]. Quantum mechanically, the difficulty is in showing that the energy distribution,  $\eta(E)$ , defined below follows an equation which implies irreversibility. Most often, in literature, this equation is taken as the Smoluchowski equation. Within the assumptions of classical linear response, this is the correct equation, as shown by Jarzynski [46]. However, our system is quantum mechanical though we have simplified our considerations somewhat by assuming that our system is adiabatic. In the two subsections below, we derive evolution equations for the energy distributions and show that these are, indeed, irreversible in time. We assume that the single-particle dynamics is non-integrable. It has been shown for chaotic quantum systems and also for non-integrable systems with zero Kolmogorov-Sinai entropy that there is level repulsion [32, 33]. That is, the probability of clustering of two adjacent levels is zero. In the quantum case, the equation contains a term in addition to the Smoluchowski equation. The subsection on semiclassical adiabatic systems shows that the quantum diffusion equation, obtained by Jain [47], reduces to the Smoluchowski equation as  $\hbar \rightarrow 0$ .

### 7.1 Quantum adiabatic systems

We consider a quantum system evolving adiabatically in time. Since the evolution is adiabatic, there is an instantaneous basis. Thus, the energy levels evolve in time as the system “deforms”. While in the same symmetry class (we assume so), the levels keep evolving without crossing. The instances when the levels come closer than the mean level spacing, there is non-zero probability of transitions to take place. For instance, very small spacing be-



tween levels then leads to an increased probability of non-adiabatic Landau-Zener transitions [51] which eventually, in long time, modify the energy distribution of the system. The basic idea behind using the Landau-Zener transitions goes back [52] in the literature of nuclear physics where it was used to argue for damping of collective modes.

Let us consider the Hamiltonian,

$$\hat{H}(t) = \hat{H}_0 + \epsilon t \hat{V} \quad (52)$$

where  $\hat{H}_0, \hat{V}$  are linear operators. We assume that the evolution in time is adiabatic which corresponds to the smallness of  $\epsilon$ .

At any instant, the system admits an eigenvalue spectrum given by the eigenvalue problem for the “frozen” Hamiltonian,

$$\hat{H}(\epsilon t)|n(\epsilon t)\rangle = E_n(\epsilon t)|n(\epsilon t)\rangle. \quad (53)$$

From time  $t = 0$ , the levels evolve in time, the resulting density operator,  $\hat{\rho}$  satisfies

$$i\hbar \frac{\partial \hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}]. \quad (54)$$

Our objective is to derive an equation for the energy distribution,

$$\eta(E) = \int^E dE' \text{tr}\{\delta(E' - \hat{H})\hat{\rho}\}. \quad (55)$$

When a system is perturbed adiabatically, there is, of course, a clear separation of time-scales. To incorporate these scales in the problem, we employ the multiple time-scale method for treating the partial differential equation (54). Accordingly, denoting the set of instantaneous states by  $\{|n(\epsilon t)\rangle\}$ , we can write an expansion for the density operator,

$$\hat{\rho}(\{|n(\epsilon t)\rangle\}, t) = \hat{\rho}_0(\{|n(\epsilon t)\rangle\}, \epsilon t) + \epsilon \hat{\rho}_1(\{|n(\epsilon t)\rangle\}, t, \epsilon t) + \dots \quad (56)$$

with the initial conditions,

$$\hat{\rho}_0 = \hat{\rho}_{00}(\hat{H}(\epsilon t)), \quad \hat{\rho}_1 = \hat{\rho}_2 = \hat{\rho}_3 = \dots = 0. \quad (57)$$

Substituting (56) in (54), we get a system of equations separated by different orders of  $\epsilon$  :

$$[\hat{\rho}_0, \hat{H}(\epsilon t)] = 0, \quad (58)$$

$$i\hbar \frac{\partial \hat{\rho}_j}{\partial t} + [\hat{\rho}_j, \hat{H}(\epsilon t)] = -i\hbar \frac{\partial \hat{\rho}_{j-1}}{\partial(\epsilon t)}, \quad j = 1, 2, \dots \quad (59)$$

If there are no other constants of the motion than  $H(\epsilon t)$  on the fast scale, or under the Thomas-Fermi approximation, by (58),

$$\hat{\rho}_0(\{|n(\epsilon t)\rangle\}, \epsilon t) = \hat{\rho}'_0(H(\epsilon t), \epsilon t) \quad (60)$$

where the arbitrariness of  $\hat{\rho}'_0$  is removed by insisting that  $\hat{\rho}$  remains valid for times  $O(\epsilon^{-1})$  by removing secularities in (59) with  $j = 1$ .

To remove the secularities in

$$i\hbar \frac{\partial \hat{\rho}_1}{\partial t} + [\hat{\rho}_1, \hat{H}(\epsilon t)] = -i\hbar \frac{\partial \hat{\rho}_0}{\partial(\epsilon t)}, \quad (61)$$

in keeping with the method, we operate by an arbitrary operator-valued function,  $g(\hat{H})$  and perform the trace of the resulting equation over the frozen basis,

$$\sum_n \left\langle n \left| g \frac{\partial \hat{\rho}_1}{\partial t} \right| n \right\rangle + \frac{1}{i\hbar} \sum_n \langle n | g[\hat{\rho}_1, \hat{H}] | n \rangle = - \sum_n \left\langle n \left| g \frac{\partial \hat{\rho}_0}{\partial(\epsilon t)} \right| n \right\rangle. \quad (62)$$

We should finally regard those results which hold for any  $g(\hat{H})$ .

For  $\hat{\rho}_0$  to be valid for times for  $O(\epsilon^{-1})$ , the right hand side (RHS) of (62) should be set to zero, which leads to

$$\sum_n \left\langle n \left| g \left[ \frac{\partial \rho'_0(\hat{H})}{\partial H} \frac{\partial \hat{H}}{\partial(\epsilon t)} + \frac{\partial \rho'_0(\hat{H})}{\partial(\epsilon t)} \right] \right| n \right\rangle = 0. \quad (63)$$

Let us re-write the density of levels as

$$\begin{aligned} \Sigma(E, \epsilon t) &= \sum_n \langle n | \delta(E - \hat{H}) | n \rangle \\ &= \frac{\partial}{\partial E} \sum_n \langle n | \Theta(E - \hat{H}) | n \rangle = \frac{\partial \Omega(E, \epsilon t)}{\partial E} \end{aligned} \quad (64)$$

where  $\Omega$  is the cumulative density of levels. In the following, it will often be useful to convert the traces into energy-integrals. For that, we define an energy average over the “frozen” Hamiltonian by

$$\frac{1}{\Sigma} \sum_n \langle n | \delta(E - \hat{H}) \dots | n \rangle = \langle \dots \rangle_{E, \epsilon t}. \quad (65)$$

With this definition, it is easy to verify that

$$\sum_n \langle n | \dots | n \rangle = \int dE \Sigma \langle \dots \rangle_{E, \epsilon t}. \quad (66)$$

Now, (63) becomes

$$\Sigma \left( \frac{\partial \rho'_0}{\partial E} \left\langle \frac{\partial \hat{H}}{\partial(\epsilon t)} \right\rangle_{E, \epsilon t} + \frac{\partial \rho'_0}{\partial(\epsilon t)} \right) = 0. \quad (67)$$

Calling

$$\left\langle \frac{\partial \hat{H}}{\partial(\epsilon t)} \right\rangle_{E, \epsilon t} = u(E, \epsilon t), \quad (68)$$

and using (66), we obtain the quantum adiabatic theorem:

$$\frac{\partial \Sigma}{\partial(\epsilon t)} + \frac{\partial}{\partial E}(\Sigma u) = 0. \quad (69)$$

In the sequel, the notation  $\partial_x$  stands for a partial derivative with respect to  $x$ .

This is a continuity equation for the level density. If the energy levels are thought of as fictitious particles, the level density is like the particle density. Indeed, in random matrix theory, one can imagine that the levels are like particles with Coulomb interaction in two dimensions [40, 55].

Therefore, (67) reduces to

$$\frac{\partial}{\partial(\epsilon t)}(\hat{\rho}'_0 \Sigma) + \frac{\partial}{\partial E}(u \Sigma \hat{\rho}'_0) = 0. \quad (70)$$

Then, for  $\hat{\rho}_0$ , we have

$$\frac{\partial \hat{\rho}_0}{\partial(\epsilon t)}(\{|n\rangle\}, \epsilon t) = \frac{\partial \hat{\rho}'_0}{\partial E}(H, \epsilon t) \left( \frac{\partial H}{\partial(\epsilon t)} - u \right). \quad (71)$$

With this equation and the initial condition (57), we have completely determined  $\hat{\rho}_0$ .

Similarly, we can proceed to determine  $\hat{\rho}_1$  [47].

Let us define the average two-time correlation function,

$$\begin{aligned} C_{\epsilon t}(s, E) &= \langle \{ \partial_{\epsilon t} \hat{H}(\{|n\rangle\}, \epsilon t) - u \} \{ \partial_{\epsilon t} \hat{H}(\{|N\rangle\}, \epsilon t) - u \} \rangle_{E, \epsilon t} \\ &= \frac{1}{\Sigma} \sum_n \left\langle n \left| \delta(E - \hat{H}) \{ \partial_{\epsilon t} \hat{H}(\{|n\rangle\}, \epsilon t) - u \} \right. \right. \\ &\quad \left. \left. \{ \partial_{\epsilon t} \hat{H}(\{|N\rangle\}, \epsilon t) - u \} \right| n \right\rangle, \end{aligned} \quad (72)$$

and a coarse grain over the energy spectrum, replacing thereby

$$g(E_n) \text{ by } \bar{g}(E), \text{ and } \left\langle n \left| \frac{\partial^2 \hat{\rho}'_0}{\partial E^2} \right| n \right\rangle \text{ by } \frac{\overline{\partial^2 \hat{\rho}'_0}}{\partial E^2}. \quad (73)$$

Also, for times of  $\mathcal{O}(\frac{1}{\epsilon})$ ,

$$\int_{-t}^0 ds C(s) = \frac{1}{2} \int_{-\infty}^{\infty} ds C(s) = \frac{1}{2} G_2. \quad (74)$$

Finally, the condition that removes secularities to  $\mathcal{O}(\epsilon^2)$  is

$$\frac{\partial}{\partial(\epsilon t)}(\hat{\rho}'_1 \Sigma) + \frac{\partial}{\partial E}(u \hat{\rho}'_1 \Sigma) - \frac{\partial}{\partial E} \left( \Sigma G_2 \frac{\overline{\partial \hat{\rho}'_0}}{\partial E} \right) - \frac{1}{2} \Sigma \frac{\overline{\partial \hat{\rho}'_0}}{\partial E} \frac{\partial G_2}{\partial E} = 0. \quad (75)$$

The energy distribution follows the following equation,

$$\frac{\partial \eta}{\partial t} = -\epsilon \frac{\partial}{\partial E}(u\eta) + \epsilon^2 \frac{\partial}{\partial E} \left[ G_2 \Sigma \frac{\partial}{\partial E} \left( \frac{\eta}{\Sigma} \right) \right] + \frac{\epsilon^2}{2} \Sigma \frac{\partial G_2}{\partial E} \frac{\partial}{\partial E} \left( \frac{\eta}{\Sigma} \right), \quad (76)$$

the quantum diffusion equation. Thus, the diffusion in quantum systems has to be qualitatively and quantitatively different as the diffusion coefficient will be different from the one we have in the Smoluchowski equation.

There are different important time-scales in the system. First of all, the time scale associated with the decay of correlation function,

$$t_c = [C(0)]^{-1} \int_{-\infty}^{+\infty} C(s) ds. \quad (77)$$

If the quantum system considered is modelled as a random matrix of dimension  $N$  [53] with large  $N$ , we know that correlation function will decay very rapidly. Thus,  $t_c$  can be very small if the quantum systems possess the following properties : (a) the number of eigenvalues is very large, and the energy spectrum is complex, and, (b) the corresponding classical system is chaotic. Chaos in the underlying classical system plays a fundamental role in the decay of correlation functions. It was shown by Gaspard and Jain [54] that the quantum time-dependent correlations in a Fermionic system are dominated by the classical correlation function. The decay of the correlation function is shown in this work to be governed by the eigenvalues of the Liouvillian operator. Thus,  $t_c$  is related to the Liapunov exponents and other detailed features of chaos. In classical ergodic adiabatic systems, the time  $t$  (fast scale) is much larger than  $t_c$ , thus the third term of (76) is zero. However, in quantal systems, we have the quantum mechanical scale,  $t_q = \hbar/\bar{\sigma}$  ( $\bar{\sigma}$  being the mean level spacing) which is why the third term at  $\mathcal{O}(\epsilon^2)$  is explicitly present. If  $t_q \ll t_c \ll t$ , the quantum effects will dominate, and all the terms in (76) will be important. If  $t_c \ll t_q \ll t$ , then the system will behave classically initially and eventually, quantum phenomena will become important; so initial evolution will be Smoluchowski-like and then non-Smoluchowski regime sets in. If, however,  $t_c \ll t \ll t_q$ , then the evolution will be according to the classical equation. Notice, as  $\hbar$  becomes small and the system is classical,  $t_q$  will become vanishingly small. In the limit, the correlations will become largely independent of energy and the third term of (76) will drop. We now show that the equation in the semiclassical limit is indeed the classical Smoluchowski equation.

## 7.2 Semiclassical adiabatic systems

The combination of semiclassical and adiabatic approximations presents a singular situation. We present here a systematic treatment where we perform an  $\hbar$ -expansion in the Wigner-Weyl basis on top of the multiple-time scale expansion. Proceeding in the same way as for the quantum adiabatic

case, we will derive an equation for the energy distribution. The density of levels is defined as

$$\Sigma(E, \epsilon t) = \sum_n \langle n | \delta(E - \hat{H}) | n \rangle. \quad (78)$$

In the Wigner-Weyl basis, it can be written through the Wigner transform of  $\delta(E - \hat{H})$  [48],

$$\begin{aligned} \delta(E - \hat{H})_W &= \delta(E - H) + \hbar^2 \left\{ -\delta''(E - H) \frac{1}{8} \frac{\partial^2 V}{\partial \mathbf{x}^2} \right. \\ &\quad \left. + \delta'''(E - H) \frac{1}{24} \left[ \left( \frac{\partial V}{\partial \mathbf{x}} \right)^2 \left( \mathbf{p} \cdot \frac{\partial}{\partial \mathbf{x}} \right)^2 V \right] \right\}. \end{aligned} \quad (79)$$

Here, the Hamiltonian is assumed to be of the form  $\frac{\mathbf{p}^2}{2} + V(\mathbf{x})$ , and the subscript  $W$  refers to the Wigner transform. The Wigner transform of  $\Sigma$  is consequently written as

$$\begin{aligned} \Sigma_W &= \int d\mathbf{p} d\mathbf{x} \delta(E - \hat{H})_W \\ &= \int d\mathbf{p} d\mathbf{x} \delta(E - H) - \frac{\hbar^2}{8} \int d\mathbf{p} d\mathbf{x} \delta''(E - H) \frac{\partial^2 V}{\partial \mathbf{x}^2} \\ &\quad - \frac{\hbar^2}{24} \int d\mathbf{p} d\mathbf{x} \delta'''(E - H) \left[ \left( \frac{\partial V}{\partial \mathbf{x}} \right)^2 \left( \mathbf{p} \cdot \frac{\partial}{\partial \mathbf{x}} \right)^2 V \right]. \end{aligned} \quad (80)$$

The primes on the Dirac delta distribution denote the weak derivatives or the distributionals [49]. The energy average of an observable,  $Q$ , is

$$\langle Q \rangle_{E, \epsilon t} = \frac{1}{\Sigma_W} \int d\mathbf{p} d\mathbf{x} \delta(E - \hat{H})_W Q. \quad (81)$$

The Wigner transform of the density operator used in the earlier sub-section is the Wigner distribution,  $f_W$ . The evolution equation for the Wigner distribution is obtained by taking the Wigner transform of the von Neumann equation :

$$\begin{aligned} \frac{\partial f_W}{\partial t} &= H \frac{2}{\hbar} \sin \left( \frac{\hbar}{2} \Lambda \right) f_W \\ &= H \left( \Lambda - \frac{\hbar^2}{24} \Lambda^3 + \dots \right) f_W. \end{aligned} \quad (82)$$

Upto  $\mathcal{O}(\hbar^2)$ , for the Hamiltonian of the form specified above, the evolution equation for  $f_W$  is

$$\frac{\partial f_W}{\partial t} = \frac{\partial V}{\partial \mathbf{x}} \cdot \frac{\partial f_W}{\partial \mathbf{p}} - \mathbf{p} \cdot \frac{\partial f_W}{\partial \mathbf{x}} - \frac{\hbar^2}{24} \frac{\partial^3 V}{\partial \mathbf{x}^3} \cdot \frac{\partial^3 f_W}{\partial \mathbf{p}^3}. \quad (83)$$

We now expand  $f_W$  in a power series of  $\hbar$  :

$$f_W = f^{(0)} + \hbar^2 f^{(2)} + \hbar^4 f^{(4)} + \dots \quad (84)$$

As a result, we obtain the following hierarchy of equations :

$$\begin{aligned}
\mathcal{O}(\hbar^0) &: \quad \partial_t f^{(0)} = \{H, f^{(0)}\}, \\
\mathcal{O}(\hbar^2) &: \quad \partial_t f^{(2)} = \{H, f^{(2)}\} - \frac{1}{24} \partial_{\mathbf{x}}^3 V \cdot \partial_{\mathbf{p}}^3 f^{(0)}, \\
&\vdots \\
\mathcal{O}(\hbar^{2n}) &: \quad \partial_t f^{(2n)} = \{H, f^{(2n)}\} - \frac{1}{24} \partial_{\mathbf{x}}^3 V \cdot \partial_{\mathbf{p}}^3 f^{(2(n-1))}.
\end{aligned} \tag{85}$$

$\{.,.\}$  denotes the Poisson bracket in the mock phase space variables. To solve these equations, we shall exploit the two time-scales,  $t$  and  $\epsilon t$ , and expand  $f^{(0)}$  ( $(\mathbf{x}, \mathbf{p}) = \mathbf{z} \in$  mock phase space [38]) as

$$f^{(0)} = f_0^{(0)}(\mathbf{z}, \epsilon t) + \epsilon f_1^{(0)}(\mathbf{z}, \epsilon t, t) + \epsilon^2 f_2^{(0)}(\mathbf{z}, \epsilon t, t) + \dots \tag{86}$$

with the initial condition,

$$f_0^{(0)}(\mathbf{z}, 0) = f_{00}, \quad f_i^{(0)} = 0 \quad \forall \quad i \geq 1 \quad \text{at} \quad t = 0. \tag{87}$$

To various orders in  $\epsilon$ , we have a hierarchy of equations nested in the  $\hbar$ -hierarchy:

$$\begin{aligned}
\mathcal{O}(\epsilon^0) &: \quad \{f_0^{(0)}, H\} = 0, \\
\mathcal{O}(\epsilon^n) &: \quad \frac{\partial f_n^{(0)}}{\partial t} + \{f_n^{(0)}, H\} = -\frac{\partial f_{n-1}^{(0)}}{\partial(\epsilon t)}.
\end{aligned} \tag{88}$$

Due to the  $\mathcal{O}(\epsilon^0)$  equation,

$$f_0^{(0)}(\mathbf{z}, \epsilon t) = f_0(H, \epsilon t) \tag{89}$$

with the initial condition,

$$f_0(E, 0) = f_{00}(E). \tag{90}$$

$f_0$  is completely specified by (90) and

$$\frac{\partial}{\partial(\epsilon t)}(\Sigma f_0) + \frac{\partial}{\partial E}(u \Sigma f_0) = 0 \tag{91}$$

removing secularities at  $\mathcal{O}(\frac{1}{\epsilon})$  in time. In (91), we have employed the following notations :

$$\begin{aligned}
\Sigma(E, \epsilon t) &= \int d\mathbf{z} \delta(E - H) = \frac{\partial}{\partial E} \Omega(E, \epsilon t), \\
\langle \dots \rangle_{E, \epsilon t} &= \frac{1}{\Sigma} \int d\mathbf{z} \delta(E - H) \dots, \\
u &= \left\langle \frac{\partial H}{\partial(\epsilon t)} \right\rangle_{E, \epsilon t}.
\end{aligned} \tag{92}$$

Eqs. (88), (91), and (92) imply

$$\frac{\partial f_0^{(0)}}{\partial(\epsilon t)}(\mathbf{z}, \epsilon t) = \frac{\partial f_0}{\partial E}(H, \epsilon t) \left( \frac{\partial H}{\partial(\epsilon t)} - u(H, \epsilon t) \right). \quad (93)$$

The formal solution of (88) for  $n = 1$  is

$$f_1^{(0)}(\mathbf{z}, t, \epsilon t) = - \int_0^t dt' \frac{\partial f_0^{(0)}}{\partial(\epsilon t)}(\mathbf{Z}, \epsilon t) + f_1(H, \epsilon t), \quad (94)$$

where  $\mathbf{Z} = \mathbf{Z}(\mathbf{z}, t, t', \epsilon t)$  is a point in mock phase space reached by starting at  $\mathbf{z}$  at time  $t$ , then evolving backward in time to  $t'$  under  $H$ .

We can determine  $f_1^{(0)}$  by removing secularities at  $\mathcal{O}(\epsilon^{-2})$  of time. Before doing so, we need some definitions which will be encountered in the sequel. The two-time correlation function is

$$C(s) = \left\langle \left( \frac{\partial H}{\partial(\epsilon t)} - u \right) \mathcal{O}_{\epsilon t}(s) \left( \frac{\partial H}{\partial(\epsilon t)} - u \right) \right\rangle, \quad (95)$$

where the operator  $\mathcal{O}$  acts to its right, evolving a point  $\mathbf{z}$  for a time  $s$  under  $H$ .  $C(s)$  satisfies the same conditions as in the previous sub-section. The condition avoiding secularities at  $\mathcal{O}(\epsilon^2)$  is

$$\frac{\partial}{\partial(\epsilon t)}(\Sigma f_1) + \frac{\partial}{\partial E}(u \Sigma f_1) - \frac{1}{2} \frac{\partial}{\partial E} \left( \Sigma G_2 \frac{\partial f_0}{\partial E} \right) = 0. \quad (96)$$

Thus, valid to  $\mathcal{O}(\epsilon)$ , the distribution function is

$$\begin{aligned} f^{(0)}(\mathbf{z}, \epsilon t) &= f_0(H, \epsilon t) + \epsilon f_1(H, \epsilon t) \\ &- \epsilon \frac{\partial f_0}{\partial E}(H, \epsilon t) \int_0^t dt' \left( \frac{\partial H}{\partial(\epsilon t)}(\mathbf{Z}, \epsilon t) - u \right), \end{aligned} \quad (97)$$

where  $f_0$  and  $f_1$  satisfy (91) and (96) respectively.

Now we expand the quantum correction,  $f^{(2)}$ , as  $f^{(0)}$  with  $\hbar$  and  $\epsilon$  as independent parameters :

$$f^{(2)} = f_0^{(2)}(\mathbf{z}, \epsilon t) + \epsilon f_1^{(2)}(\mathbf{z}, \epsilon t, t) + \epsilon^2 f_2^{(2)}(\mathbf{z}, \epsilon t, t) + \dots \quad (98)$$

To orders in  $\epsilon$ , we have

$$\begin{aligned} \mathcal{O}(1) &: \{H, f_0^{(2)}\} = \frac{1}{24} \partial_{\mathbf{x}}^3 \cdot \partial_{\mathbf{p}}^3 f_0^{(0)}, \\ \mathcal{O}(\epsilon^i) &: \frac{\partial f_i^{(2)}}{\partial t} = \{H, f_i^{(2)}\} - \frac{\partial f_{i-1}^{(2)}}{\partial(\epsilon t)} - \frac{1}{24} \partial_{\mathbf{x}}^3 \cdot \partial_{\mathbf{p}}^3 f_i^{(0)}, \dots \end{aligned} \quad (99)$$

The solution of (99) (for  $f_0^{(2)}$ ) is [64]

$$f_0^{(2)}(\mathbf{z}, \epsilon t) = \frac{1}{8} \left\{ -\partial_{\mathbf{x}}^2 V \partial_E^2 f_0^{(0)} - \left[ \frac{1}{3} (\partial_{\mathbf{x}} V)^2 + \frac{1}{3} (\mathbf{p} \cdot \partial_{\mathbf{x}})^2 V(\mathbf{x}) \right] \partial_E^3 f_0^{(0)} \right\}. \quad (100)$$

Upto  $\mathcal{O}(\epsilon)$  and  $\mathcal{O}(\hbar^2)$ , the semiclassical density is

$$\begin{aligned}
f_W(\mathbf{z}, \epsilon t, t) &= f_0(h, \epsilon t) + \epsilon f_1(h, \epsilon t) - \epsilon \frac{\partial f_0}{\partial E}(h, \epsilon t) \\
&\quad \int_0^t dt' \left[ \frac{\partial h}{\partial(\epsilon t)}(Z, \epsilon t) - u(h, \epsilon t) \right] + \frac{\hbar^2}{8} \left\{ -\partial_{\mathbf{x}}^2 V \partial_E^2 f_0^{(0)} \right. \\
&\quad \left. - \left[ \frac{1}{3}(\partial_{\mathbf{x}} V)^2 + \frac{1}{3}(\mathbf{p} \cdot \partial_{\mathbf{x}})^2 V(\mathbf{x}) \right] \partial_E^3 f_0^{(0)} \right\}. \quad (101)
\end{aligned}$$

Now we consider (99). Multiplying by a test function,  $g(h)$  in the equation for  $i = 1$  and integrating over the phase space gives us

$$\frac{\partial}{\partial t} \int d\mathbf{z} g(h) f_1^{(2)} = - \int d\mathbf{z} g(h) \frac{\partial f_0^{(2)}}{\partial(\epsilon t)} - \frac{1}{24} \int d\mathbf{z} g(h) \partial_{\mathbf{x}}^3 V \cdot \partial_{\mathbf{p}}^3 f_1^{(0)}. \quad (102)$$

Since the RHS is independent of  $t$ , the LHS grows secularly. We set the RHS to zero, and after some manipulations, we get

$$\frac{\partial}{\partial(\epsilon t)} (\Sigma f_0^{(2)}) + \frac{\partial}{\partial E} (u \Sigma f_0^{(2)}) + \int dE \Sigma \left\langle \frac{1}{24} \partial_{\mathbf{x}}^3 V \cdot \partial_{\mathbf{p}}^3 f_1^{(0)} \right\rangle_{E, \epsilon t} = 0. \quad (103)$$

With (91), (96), and (103), we obtain the semiclassical diffusion equation,

$$\begin{aligned}
\frac{\partial \eta}{\partial t} = & - \epsilon \frac{\partial}{\partial E} (u \eta) + \frac{1}{2} \epsilon^2 \frac{\partial}{\partial E} \left[ G_2 \Sigma \frac{\partial}{\partial E} \left( \frac{\eta}{\Sigma} \right) \right] \\
& - \hbar^2 \int dE \Sigma(E, \epsilon t) \left\langle \frac{1}{24} \partial_{\mathbf{x}}^3 V \cdot \partial_{\mathbf{p}}^3 f_1^{(0)} \right\rangle_{E, \epsilon t}. \quad (104)
\end{aligned}$$

We have not succeeded to include  $f_1^{(2)}$  in this work.

Note that (104) reduces to the Smoluchowski equation as  $\hbar \rightarrow 0$ .

## 8. Geometric phase and dissipation

Adiabatic approximation leads to linear response theory on one hand where dynamical susceptibility (or polarization propagator) is central, and geometric phases on the other. When a particle (e.g., a nucleon) moves inside an enclosure whose boundary is adiabatically vibrating in time, the wave-function can acquire a geometric phase over a cycle of vibration. In this section, following [15], we show that the geometric phase thus acquired is related to the imaginary part of susceptibility. It is well-known that the imaginary part of susceptibility is a quantifier of dissipation, geometric phase plays an important role in understanding of damping of collective excitations.

To understand how this relation comes about, for the sake of brevity, we restrict ourselves to the case of cyclic evolution of deformations of the many-body system. Let us denote the Hamiltonian parametrised by some parameters (e.g., deformation parameters),  $\mathbf{R}$ , by  $H(\mathbf{R})$  which describes a single particle in an effective mean-field. It is well-known [59] that when



the parameters evolve along a cyclic path  $\mathcal{C}$  the instantaneous eigenfunction of the system  $|n(\mathbf{R})\rangle$  corresponding to the eigenvalue,  $E_n(\mathbf{R})$ , acquires a geometric phase given by

$$\begin{aligned}\gamma_n(\mathcal{C}) &= \oint_{\mathcal{C}} i \langle n(\mathbf{R}) | \nabla_{\mathbf{R}} n(\mathbf{R}) \rangle \cdot d\mathbf{R} \\ &= -\frac{1}{\hbar} \int_{\mathcal{S}} \mathbf{V}_n \cdot d\mathbf{S},\end{aligned}\quad (105)$$

where  $\mathcal{S}$  is the surface enclosed by  $\mathcal{C}$  in the parameter space, and  $\mathbf{V}_n$  is the “field strength” (adiabatic curvature) given by a familiar expression involving a wedge product :

$$\mathbf{V}_n = -i\hbar \sum_{m(\neq n)} \frac{\langle n | \nabla_{\mathbf{R}} H | m \rangle \wedge \langle m | \nabla_{\mathbf{R}} H | n \rangle}{(E_n - E_m)^2}. \quad (106)$$

A suitable form of  $\mathbf{V}_n$  for the sequel is [61]

$$\mathbf{V}_n = \frac{i}{2\hbar} \lim_{\epsilon \rightarrow 0} \int_0^\infty dt e^{-\epsilon t} \langle n | [(\nabla_{\mathbf{R}} H)_t, \wedge (\nabla_{\mathbf{R}} H)] | n \rangle \quad (107)$$

where  $(\nabla_{\mathbf{R}} H)_t$  denotes the Heisenberg-evolved operator. Note that, the state  $|n(\mathbf{R})\rangle$  appearing in (105) corresponds to a single-particle eigenket in an effective mean-field. This state is clearly related to the original many-body Fermi system for which the imaginary part of the dynamical susceptibility is [34, 60]

$$\chi''(t) = \frac{1}{2\hbar} \langle \Phi_0 | [\hat{\mathcal{A}}(t), \hat{\mathcal{B}}(0)] | \Phi_0 \rangle = \int \frac{d\omega}{2\pi} e^{-i\omega t} \tilde{\chi}''(\omega) \quad (108)$$

where  $|\Phi_0\rangle$  is the pure ground state of the many-particle system with Fermi energy,  $E_F$ . If one-body operators,  $\hat{H}$ ,  $\hat{A}$ , and  $\hat{B}$ , are used to construct many-body operators by a direct sum so as to get  $\hat{\mathcal{H}}$ ,  $\hat{\mathcal{A}}$ , and  $\hat{\mathcal{B}}$ , respectively, and  $\hat{\mathcal{H}}|\Phi_l\rangle = \mathcal{E}_l|\Phi_l\rangle$ , then we [54] have

$$\langle \Phi_0 | \hat{\mathcal{A}} | \Phi_l \rangle = \langle m(\mathbf{R}) | \hat{A} | n(\mathbf{R}) \rangle. \quad (109)$$

On reducing the many-body system at  $T = 0K$ , (where the Fermi-Dirac distribution is a Heaviside step function), to one-body system, we can express [54]

$$\tilde{\chi}_{AB}''(\omega) = -\frac{\omega}{2} \int dt e^{i\omega t} \text{tr} \delta(E_F - H) [\hat{A}(t), \hat{B}(0)]. \quad (110)$$

This can be written as in Sec. 4 semiclassically.

The label  $n$  in (105) corresponds to single-particle states and is related to  $|\Phi_0\rangle$  because the many-body matrix elements can be written in terms of one-body matrix elements for the case when all the constituents are taken as non-interacting. In many-body physics, this gives the zero-order response whereupon the interaction can be included in a Vlasov description in an

iterative way [63]. For relating the response function to the geometric phase, the operators  $\hat{\mathcal{A}}$  and  $\hat{\mathcal{B}}$  in our discussion are to be identified with  $\nabla_X \hat{\mathcal{H}}$  and  $\nabla_Y \hat{\mathcal{H}}$  for  $\mathbf{R} = (X, Y, Z)$ .

The matrix element in (105) can be written as a many-body matrix element using (109) by composing  $\hat{A}$  and  $\hat{B}$  so that we get the operator,  $\mathcal{C}(t) = [\mathcal{A}(t), \mathcal{B}(0)] - [\mathcal{B}(t), \mathcal{A}(0)]$ , which is related to a difference  $\chi''_{AB}(t) - \chi''_{BA}(t) = \chi''_C(t)$ . Thus, we can re-write  $\mathbf{V}_n$  as

$$\begin{aligned} \mathbf{V}_n &= \frac{i}{2\hbar} \lim_{\epsilon \rightarrow 0} \int_0^\infty dt e^{-\epsilon t} t \langle \Phi_0 | \mathcal{C} | \Phi_0 \rangle \\ &= \int_0^\infty dt t \chi''_C(t) = - \left. \frac{\partial \tilde{\chi}''(\omega)}{\partial \omega} \right|_{\omega=0}. \end{aligned} \quad (111)$$

We now arrive at our relation for the case of cyclic evolution :

$$\gamma_n(\mathcal{C}) = \int_{\mathcal{S}} d\mathbf{S} \cdot \left. \frac{\partial \tilde{\chi}''_C(\omega; \mathbf{R})}{\partial \omega} \right|_{\omega=0}. \quad (112)$$

Since  $\tilde{\chi}''_C(\omega)$  is related to energy dissipation, this relation connects geometric phase to dissipation. In other words, we see that dissipation in finite systems is purely quantum mechanical as it is related to geometric phase. That this forms a fundamental basis for the Bohr's liquid drop model is discussed elsewhere [15].

Similarly, we can find a relation for the case of non-cyclic evolutions.

It is known that friction or viscosity in a quantal system can appear in a thermodynamic limit. Since we have a many-body system, seemingly contradictory conditions of finiteness of the system and “continuity” of the energy spectrum are met with. The finite size explicitly manifests itself in terms of a sum over periodic orbits.

It is important to note that the wavefunction that we have considered above is the one which satisfies the requirements in Section 3.

It is interesting to note that the viscosity of quantum Hall fluid in two dimensions at zero temperature is related to adiabatic curvature [62]. Also, for covalent dielectrics, the polarisation is related to geometric phase. These two examples and our general treatment leads us to think of a deeper unified connection.

## 9. Fission viscosity tensor

Since long, linear response theory is used to study the dynamics of fission process [65] in nuclei. The collective coordinates are the deformation parameters and pairing gap. The quantity related to fission viscosity tensor is the first moment of the time-dependent response function. In this paper, we have presented semiclassical and random matrix expressions for the response function. In this Section, we apply this knowledge to obtain expression for viscosity tensor in terms of periodic orbits of the underlying classical

system. It will also turn out that the classical Liouvillian operator plays a fundamental role. This is very interesting as the eigenvalues of the Liouvillian dictate the decay of correlations. Thus, the existence of dissipation is directly related to the non-integrability of the classical system.

If we denote by  $\{q_\nu\}$  the set of  $N$  collective coordinates, then the friction viscosity tensor is given by

$$\gamma_{\nu\mu} = {}^{(1)}M_{\nu\mu}, \quad (113)$$

where  ${}^{(n)}M_{\nu\mu}$  are the moments of the time-dependent response function,  $C_{\nu\mu}(t, T)$ . The system is at a temperature,  $T$ , related to the excitation energy of the intrinsic system. It was brought into the calculations when the intrinsic degrees of freedom were averaged over the canonical ensemble. Once again, we recall that the canonical statistical mechanics follows from the assumption of microscopic chaos in single-particle motion.

The semiclassical form of the time-dependent response function (or the time correlation function) gives the following expression for the viscosity tensor when we assume that the single-particle-dynamics is fully chaotic (in case the dynamics is mixed, expression can be immediately written using (51)) :

$$\begin{aligned} \gamma_{\nu\mu} &= \int_0^\infty dt \, t C_{\nu\mu}(t, T) \\ &= \frac{1}{Z(\beta)} \int \frac{d^f x d^f p}{(2\pi\hbar)^f} e^{-\beta H_{cl}} \int_0^\infty dt \, t \left( \frac{\partial H}{\partial q_\nu} \right)_W \exp(-\hat{\mathcal{L}}_{cl}) \left( \frac{\partial H}{\partial q_\mu} \right)_W \\ &+ \frac{1}{\pi\hbar Z(\beta)} \sum_p \sum_{r=1}^\infty \int dE e^{-\beta E} \frac{\cos\left(\frac{r}{\hbar} S_p(E) - r\frac{\pi}{2}\nu_p\right)}{|\det(\mathbf{m}_p^r(E) - \mathbf{I})|^{1/2}} \\ &\int_0^\infty dt \, t \oint_p d\tau \left( \frac{\partial H}{\partial q_\nu} \right)_W(\tau) \left( \frac{\partial H}{\partial q_\mu} \right)_W(\tau + t). \end{aligned} \quad (114)$$

This is a semiclassical expression based on incorporation of the Gutzwiller trace formula in the linear response theory. Applications to the practical situations in nuclear fission will be of a great interest.

A random matrix expression for the viscosity tensor can also be readily written employing the results of Section 5. The coefficient of friction in the zero-frequency limit is

$$\gamma_0 = - \left. \frac{\partial \tilde{\chi}''(\omega)}{\partial \omega} \right|_{\omega=0} \quad (115)$$

where  $\tilde{\chi}''(\omega)$  is given by (40). Notice that friction is physically the adiabatic curvature, from (112). This has been also found earlier for viscosity of quantum Hall systems [62].

## 10. Summary

A lot of work has been done on the nature of nuclear dissipation (see, e.g., [68]). It has been stated, most clearly by Weidenmüller [9], that chaos in the nucleonic motion will be related to dissipation. However, before trying to explain the experimental data, we need a systematic theory. In this work, we have taken a step in realising this connection.

On the basis of the connection established between quantum chaos and statistical mechanics, we have developed semiclassical and random matrix theory of response to external perturbations of a many-body Fermi system. For the cases where the dynamics of a single particle is chaotic or not fully chaotic but mixed, we have presented the results for response functions. Also, we have shown that the collective excitations damp irreversibly and this irreversibility of damping is shown to be quantum mechanical. The damping of collective motion is reflected in the re-distribution of energy. Energy re-distribution has been shown to evolve irreversibly. The reason for this irreversibility is the complexity of the energy spectrum which might, in turn, be connected with classical non-integrability (chaos or mixed dynamics). The complexity is such that, in the scaling limit of the energy spectrum, the system behaves as if it is a realisation of an ensemble of random matrices. Quantum chaos or complexity of spectrum brings with itself a kind of almost periodic evolution so that the system takes much longer to show any recurrence. The recurrence is a must for a system with a discrete spectrum. However, collective excitations decay much before this recurrence can occur. To present a simple instructive example, we consider the set of prime numbers and look at

$$\mathcal{P}(t) = \sum_{\text{primes}, p} \exp[ipt], \quad (116)$$

which can be thought of as the survival probability (a special case of two-time correlation function) of a fictitious system. This sum decays quickly as the number of primes increase. Clearly then, for a many-body system like a nucleus where the spectrum is quite complex, the time correlations decay very quickly. This will be decided by the spectral properties of the Liouvillian [28]. The coarse-graining we had done in Sec. 7 was a representative of the complexity we have discussed here.

Another point of importance is the arbitrariness of the initial density operator. We have obtained quantum and semiclassical generalisations of the Smoluchowski equation. Damping is described in terms of the imaginary part of the response function. This is shown to be related to geometric phase acquired by a single-particle wavefunction as the many-body system evolves.

The application of the general theory based on existence of chaos or mixed chaos in the many-body system is being applied to fission of nuclei and metallic clusters. We have presented an explicit expression for the fission viscosity tensor in Sec. 9 which can be used to analyse the fission data for relatively heavier nuclei. Clearly, the quantum diffusion equations obtained

here are directly applicable to fission processes [56, 57] where the Smoluchowski equation has been used until now [58]. The difference new term will make in the assignment of transport coefficients is of interest. In case of nuclei, in the past [67], there has been a criticism in modelling the deforming mean-field by the form,  $\hat{Q}F^{\text{ext}}(t)$ . However, this is basically a matter of convenience. However, while making realistic calculations, one may keep this in mind. The advantage one has here is that one gets the response function.

With the semiclassical wall formula obtained here, we are now trying to implement it by finding periodic orbits of the underlying classical system. Since the trace formulae can be written in terms of zeta functions, the numerical implementation will be efficient. Hopefully, not too many periodic orbits will be needed. In this regard, since the deformed nuclei can be modelled in terms of harmonic oscillator potentials, and we have the exact trace formulae for them [66], it will be interesting to use our expressions to find response functions.

We hope to study nonlinear response in a semiclassical and random-matrix setting in future. This is important for understanding phenomena at an ultra-fast time scale.

Hence, we have shown that there is an emergence of a new understanding of damping of collective excitations, on quantum transport coefficients in finite Fermi systems, and in the general theory of response of nuclei, metallic clusters, and quantum dots.

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